Singularity-Avoidance Control of Robotic Systems with Model Mismatch and Actuator Constraints

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Motivation

Challenges with singularities:

Multi-link robots
Singularities may result in the loss or gain of one or more DoFs, potentially leading to a loss of control over the system.

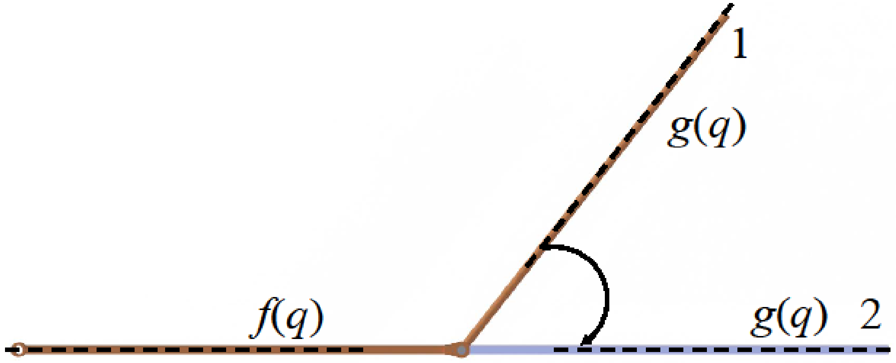
Actuator constraints
Limited actuation in real systems

Model mismatch
Inaccuracies in the dynamic model compared to the physical system

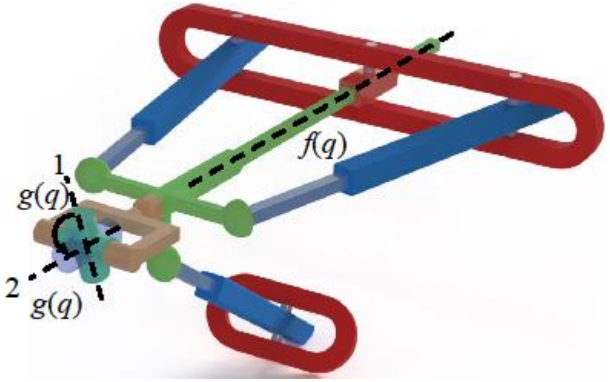
Contribution:

Control barrier function-based Singularity-Avoidance Control

Explicitly design methods subject to actuator constraints and model mismatch



DoF serial manipulator



5- DoF hybrid manipulator

Control Barrier Functions - Basics

Nonlinear affine system: $\dot{x} = f(x) + g(x)u$

Safe set (constraint set) $\mathcal{C} := \{x \in \mathbb{R}^n : h(x) \geq 0\}$

$h(x)$: barrier function

Forward invariance

If the system starts in $\mathcal{C}$, $x(0) \in \mathcal{C}$ and $x(t) \in \mathcal{C}\ \forall t \geq 0$

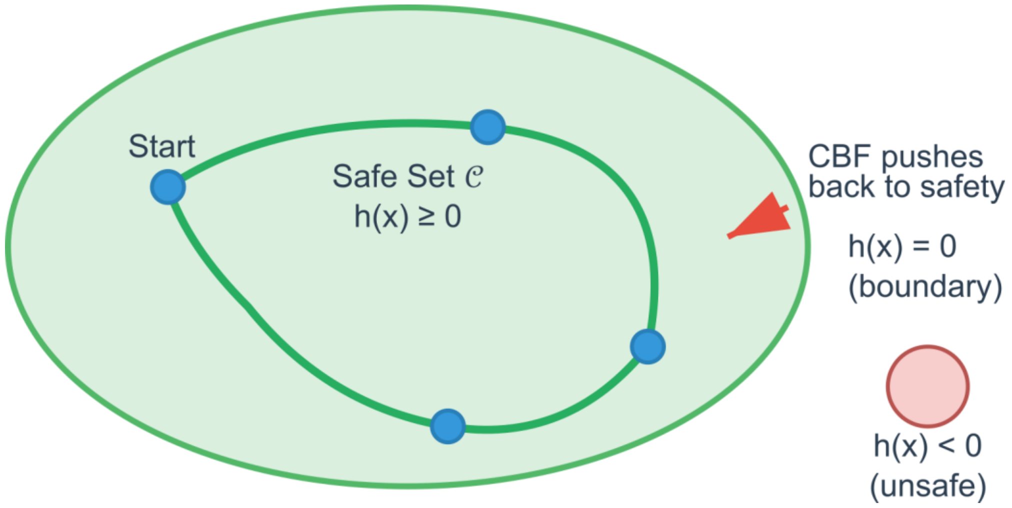
CBF condition

$$\sup_{u \in \mathcal{U}} [L_f h(x) + L_g h(x) u + \alpha(h(x))] \geq 0$$

Natural dynamics

Control effect

Safety margin: $\alpha(h(x))$ is an extended class $\mathcal{K}$ function



Find a control input u that satisfies the CBF condition!

Problem Formulation

Goal: Avoid singularities in the presence of mismatch in the control model, and in the presence of disturbances

Robotic system dynamics

$$\dot{q} = v$$

$$\dot{v} = M(q)\,(u - C(q,v)v - G(q) - d(x))$$

Intertia matrix — Centrifugal force — Gravity — Mismatch (unknown)

Model mismatch $d(x) = [d_1(x), \dots, d_n(x)]^T$

angular positions and velocities $x = [q, v]^T$

Assumptions:

1. We can model the mismatch using a kernel-based method with a bound on the RKHS norm.
2. The system state q is bounded by hard constraints, i.e., $q \in [q_{max}, q_{min}]$ and $q_{max} = -q_{min}$.

Properties:

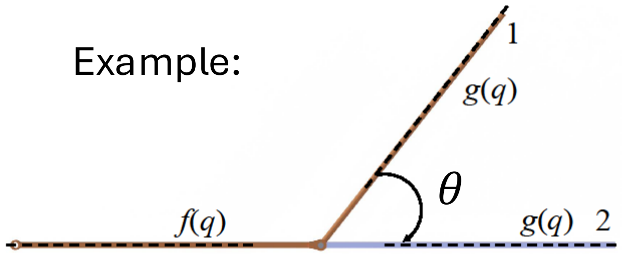
- $M(q)$ is a symmetric, positive definite matrix
- There exist c_{max} and $g_{max} \in \mathbb{R}_{>0}$ such that $C(q,v) \leq c_{max}\|v\|$ and $\|G(q)\| \leq g_{max}$.

Constraints and control problem

Singularity: $f(q)$ and $g(q)$ are parallel, $\arccos(f(q)^T g(q)) = 0$

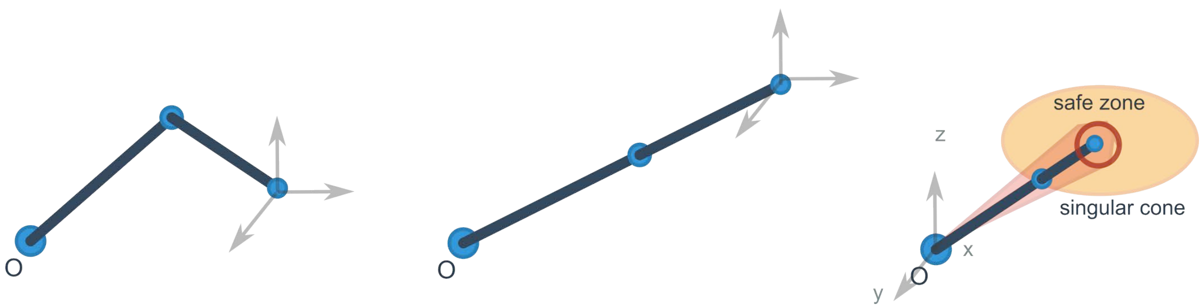
Singularity cone: $z(q) \coloneqq 1 - \varepsilon - f(q)^T g(q)) \geq 0$

Example:



$\eta(q) = f(q)^T g(q) = \cos(\theta)$

Normal Configuration | Singular Configuration | Singularity Cone



Singularity constraint: $\mathcal{Z} := \{q \in \mathbb{R}^n : z(q) \geq 0\}$

Velocity (state) constraints: $\mathcal{V}_i := \{v_i \in \mathbb{R} : \overline{b}_i(v_i) \geq 0, \underline{b}_i(v_i) \geq 0\}, \forall i \in \mathbb{N}_n,$
where $\overline{b}_i \coloneqq v_{max} - v_i$ and $\underline{b}_i \coloneqq v_i - v_{min}$

Input (actuator) constraints: $\mathcal{U} := \{u_i \in \mathbb{R}, \forall i \in \mathbb{N}_n : u_{max} - u_i \geq 0, u_i - u_{min} \geq 0\}$

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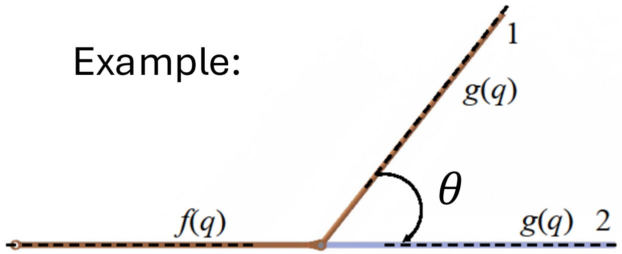

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Goal:

Given the robotic system
$$\dot{q} = v$$
$$\dot{v} = \dot{M}(q)\big(u - C(q, v)v - G(q) - d(x)\big)$$
$$x = [q, v]^T$$
subject to model mismatch $d(x)$ and actuator constraints $\mathcal{U}$,
design a control strategy $u \in \mathcal{U}$
such that the singularity constraint $\mathcal{Z}$, and the velocity constraints $\mathcal{V}_i$ are forward invariant.

CBF condition:

$$\sup_{u\in\mathcal{U}}[L_f h(x) + L_g h(x)u + \alpha(h(x))] \geq 0$$

Relative degree issue:

Condition for forward invariance: $\dot{z}(q) \geq -\alpha(z(q))$

u has no effect on $\dot{z}(q)$: $\dot{z}(q) = -(\partial\eta/\partial q)^T v$

Define $\boldsymbol{h}(\boldsymbol{x}) = \dot{\boldsymbol{z}}(\boldsymbol{q}) + \boldsymbol{\gamma}\boldsymbol{\beta}_1(\boldsymbol{z}(\boldsymbol{q}))$, and $\mathcal{C} := \{x \in \mathbb{R}^n : h(x) \geq 0\}$

Safety control implementation:

$$\min_{u\in\mathcal{U}} \| u - u_{nom} \|^2$$

$$\text{s.t.} \quad \dot{h}(x) \geq -\delta\beta_2(h(x))$$

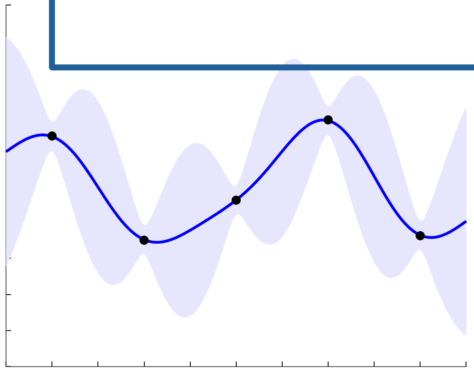
$$\dot{\overline{b}}_i(v_i) \geq -k\beta_3\left(\overline{b}_i(v_i)\right)$$

$$\dot{\underline{b}}_i(v_i) \geq -k\beta_3(\underline{b}_i(v_i))$$

Goal:

Design parameter γ, δ and k such that there always exists a feasible solution $u^* \coloneqq \underset{u\in\mathcal{U}}{\operatorname{argmin}} \| u - u_{nom} \|^2$ that satisfies all constraints.

Lemma. Given **Assumption** 1, and a training dataset $\mathcal{D} := \{(x_i, y_i)\}_{i=1}^{M}$, the prediction error of GP is bounded by $\| \mu(x) - d(x) \| \leq \lambda(x) := \sqrt{\sum_{i=1}^{n} (B_i^2 - \omega_i + M)\sigma_i^2}$.



GP uncertainty (posterior variance)

B_i^2: Physical magnitude bounds (increasess uncertainty)

ω_i: Information gain (reduces uncertainty)

M: Safety margin (adds conservatism)

State-independent bound (more conservative)

$$\lambda(x) \leq \bar{\lambda} := \sqrt{\sum_{i=1}^{n} (B_i^2 - \omega_i + M) \max_{\substack{v_i \in \mathcal{C} \cap \mathcal{V}_i \forall i \in \mathbb{N}_n \\ q \in \mathcal{Z} \cap \mathcal{C}}} k_i(x, x)}$$

Forward invariance

Assumption: 3. The system has sufficient control effort (authority)

such that $u_{max} \geq \frac{\xi}{\sqrt{3}\eta_{qmax}m_{max}}$.

ξ:max total disturbance

the control should overcome all the forces + model uncertainties

➢ render $\mathcal{C}$ forward invariant

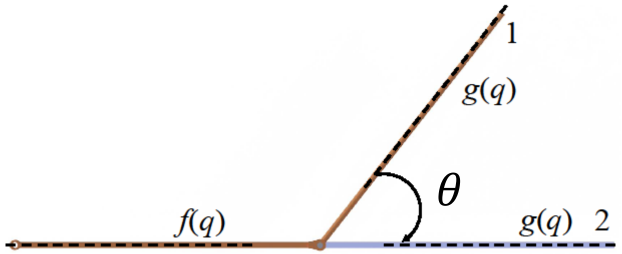
Theorem 1: if

$$-\Gamma^T M(\mathrm{q})^{-1}(u - C(q,v)v - G(q) - \mu(x)) - \gamma \frac{\partial \beta_1}{\partial z}\Gamma^T v - v^T\left(\frac{\partial^2 \eta}{\partial q^2}\right)^T v \geq -\delta\beta_2(h(x)) + \| \Gamma^T M(q)^{-1} \| \bar{\lambda}.$$

Condition 1

then: the input u renders $\mathcal{C}$ forward invariant.

$$\Gamma = \frac{\partial \eta}{\partial q}$$



$$\eta(\mathrm{q}) = \mathrm{f}(\mathrm{q})^{\mathrm{T}}\mathrm{g}(\mathrm{q}) = \cos(\theta)$$

Forward invariance

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Condition 1

then: the input u renders $\mathcal{C}$ forward invariant.

Lemma 1: if

$$3\eta_{max^2} v_{max}^2 + \sqrt{3}\gamma \frac{\partial \beta_1}{\partial z}\eta_{max} v_{max} - \delta\beta_2(h(x)) + \eta_{qmax} m_{max}(\sqrt{3}u_{max} + c_{max} v_{max}^2 + g_{max} + \bar{\lambda} + \| \mu(x) \| \leq 0$$

Then: there always exists $u \in \mathcal{U}$ that enforces condition 1.

Condition 2

The same way we can prove forward invariance for the velocity constraint.

Singularity condition:

$$f(q) = [\cos q_1 , \sin q_1 , 0]^T$$
$$g(\mathrm{q}) = [\cos q_1 + q_2 + q_{ini} , \sin q_1 + q_2 + q_{ini} , 0]^T$$

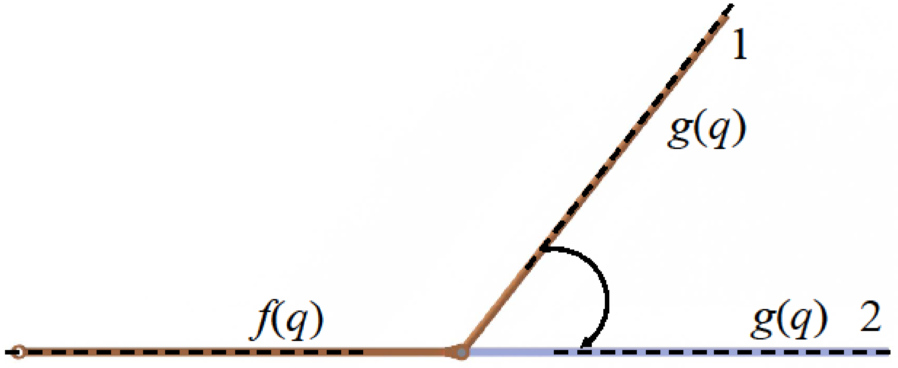
Nominal control: PID controller

$$u_{nom} = k_p e + k_i \int_0^t e \, d\tau + k_d \dot{e}$$

Safety control law:

$$u^* \coloneqq \underset{u \in \mathcal{U}}{\operatorname{argmin}} \parallel u - u_{nom} \parallel^2$$
$$\text{s.t. } F(u, q, v) \geq 0$$

(Condition 1 and Condition 2)



Simulations: High-fidelity simulation of a 2-DoF planar manipulator in Simscape

Input: $u_{max} = -u_{min} = 5, q_{max} = -q_{min} = \pi/3, v_{max} = -v_{min} = 2.$

unknown lumped mass m=0.2 kg.

GP setting: $k_i = 0.01^2 \exp(-\| x_1 - x_2 \|^2/2)$, M = 200

Choose extend class-K function:

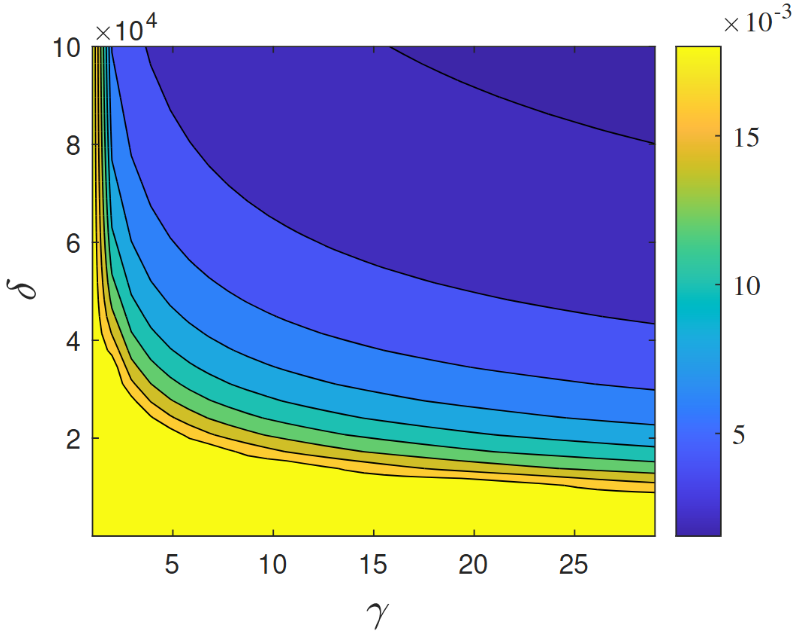
$$\beta_1(z) = z, \beta_2(h) = h^3$$

$$\beta_3(\bar{b}_i) = \tan^{-1}(\bar{b}_i) \text{ (i.e., } \beta_3(\underline{b}_i) = \tan^{-1}(\underline{b}_i))$$

Calculate γ^* and choose $\gamma \leq \gamma^*$

Calculate δ^* and choose $\delta \geq \delta^*$

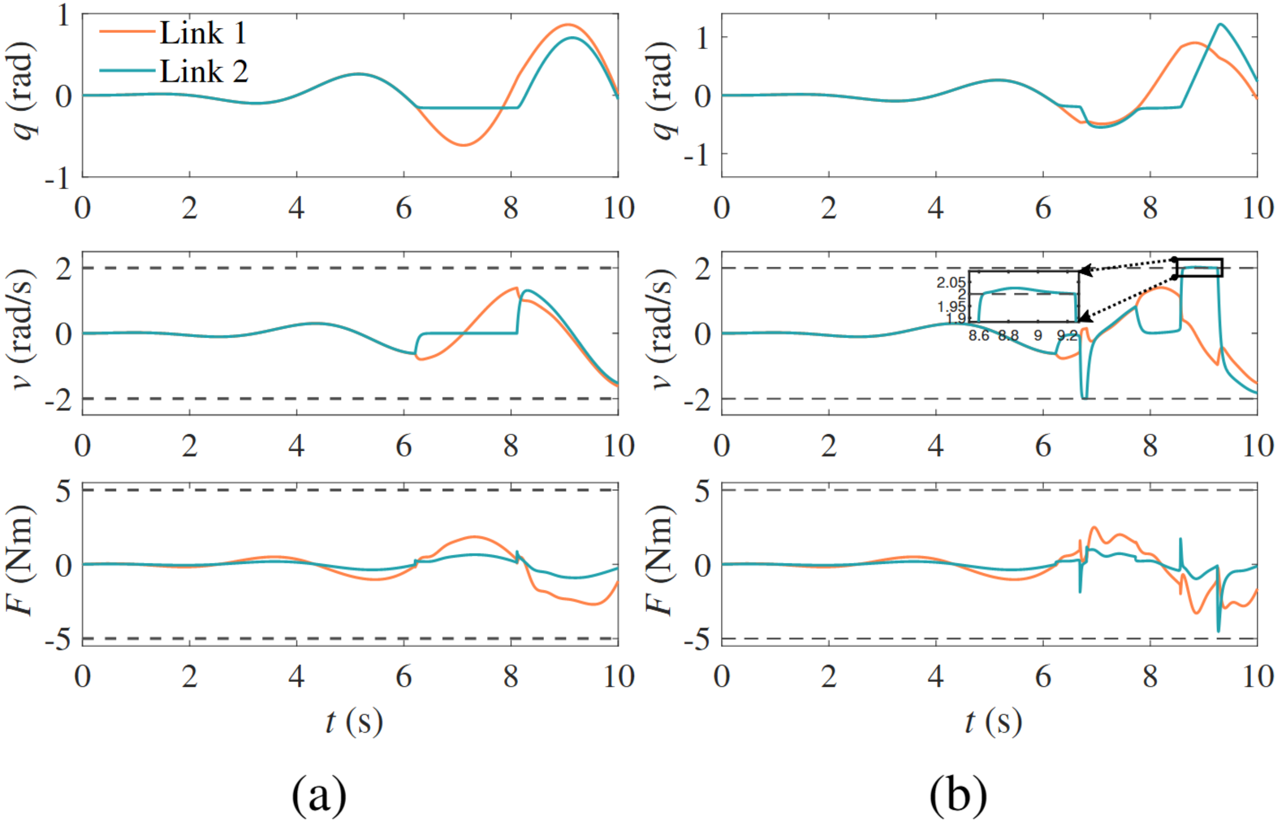
CBF conservatism trade-off



The impact of γ and δ on the singularity constraint $z_{min}(q)$

Simulations: High-fidelity simulation of a 2-DoF planar manipulator in Simscape

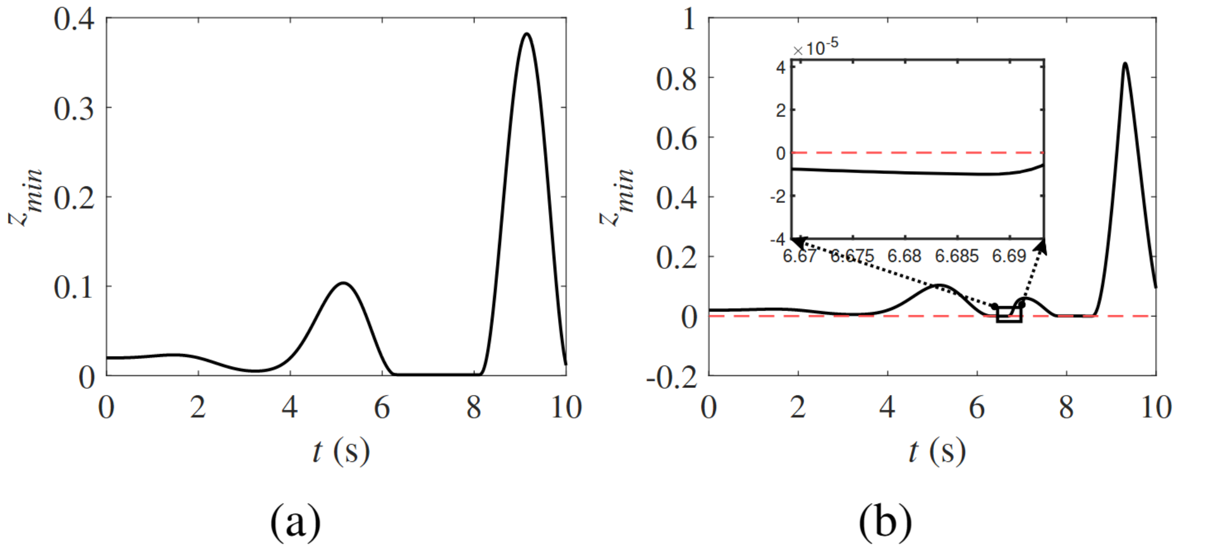
Velocity constraint violation!



Comparison of trajectory tracking results. (a) with GP regression. (b) without GP regression.

$\gamma = 29$
$\delta = 100000$

Singularity configuration!



Comparison of singularity constraints.
(a) with GP regression. (b) without GP regression.

Conclusion and Future Work

We proposed singularity-avoidance control law for robotic systems subject to model mismatch and actuator constraints.

CBF parameter ensures the feasibility of the optimization problem under actuator constraints.

Velocity constraints are also considered to ensure safety.

Future work:

Universal method that encompasses all singularity configurations

Potential conflict between the singularity constraint and velocity constraint should be addressed